A Family of Skew-unimodal Distributions with Mode Invariance

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Abstract

There have been many studies on a skew-symmetric distribution. It consists of an underlying symmetric density and a perturbation factor to control the skewness. The skew-symmetric distribution is very flexible, but not necessarily unimodal even if the underlying symmetric distribution is unimodal. It is often a hard task to investigate whether a new density is unimodal. To get relief from this task, a skew-unimodal distribution has been discussed recently in which the normalizing constant remains unchanged using a specific class of transformation of scale to control the skewness. This paper incorporates the mode invariance into a skew-unimodal distribution. The mode remains unchanged for any transformation. The monotonicity of skewness can be shown under a weak condition. The condition obtained here is very simple by virtue of mode invariance and allows us to incorporate various favorable properties into a skew-unimodal distribution.